



Teltest Electronics Laboratories
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On Maxwell's *Special* Theory Of Electromagnetism

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James Clerk Maxwell in his “A Treatise on Electricity and Magnetism” (henceforth Treatise)^{1,2} put forth his theory of electromagnetism that has empowered the world of electromagnetics to greater heights for the last century and a half. For the purposes of this paper, I will call that theory Maxwell's *Special Theory of Electromagnetism*. Just suppose there would exist a Maxwell's *General Theory of Electromagnetism* and it is at our finger tips. (More on that toward the end of this paper.)

First, a little about myself, and my reasons for engaging in my project of deep diving into the source of Maxwell's equations, that have led to this paper. I have begun the translation of the Equations portion of Maxwell's Treatise to modern notation for a number of reasons. Primarily, I have hungered for years to understand the origins and meaning of Maxwell's Equations, or better stated, the Maxwell/Heaviside-Gibbs Equations³. I have been a licensed Professional Engineer, an iNarte certified EMC Engineer, am a Life Senior member of the IEEE, and have been an active Ham radio operator for 70 years. I have taken a number of graduate academic courses on Advanced Engineering Electromagnetics and from what I can see now, that those courses, as well as our practice of electromagnetic engineering, consisted of exploring the *fruit* of Maxwell's Equations, not the pure science of Maxwell's Equations. Even though, I have had a long career in Electronic Engineering, a good deal of which was successfully dealing in-depth with electromagnetics, yet, still the complete in-depth understandings or even remembering Maxwell's Equations eluded me.

Over the last 20 years or so, a number of people have asked me of my opinion about a subject that has been bantered about relating to the production of “*free energy*” and longitudinal electromagnetic waves that are derived from the application of Maxwell's equations in quaternion form (Discussed below). I did have some familiarity with quaternions before I began this pursuit. I had implemented quaternions to perform rotations in the data processing of a strap-down inertial platform (In the handle of a golf club, of all things). However, that had been basically a cookbook implementation and I was really not on-top-of quaternion algebra. I had read in a number of places that Maxwell in his Treatise had expressed his equations in quaternions. And unfortunately, at the insistence of his edi-



tors and publishers, in subsequent editions, had removed or at least down-played reference to them.

In any case, my knowledge at the time regarding Maxwell's equations and his Treatise was very limited, almost non-existent, and I had to reply to those asking, "that I simply did not know". I wanted to know; however, I could not find a path to that knowledge, even though I did enroll at UT as a doctoral student and took a number of courses over two years in electrical engineering including advanced engineering electromagnetics. Although I could work with the fruit of Maxwell's equations quite well, I really did not understand Maxwell's equations in the way I would like. Over the years I have made a number of stabs at remedying this, including acquiring quite a library on the subject, all to no avail. And then -----

After reading a couple of delightful books on Faraday, Maxwell and Heaviside^{4,5}, I decided to dive into it more deeply. I obtained copies of both volumes of his Treatise and started my study. My primary interest was in the section of Volume II dealing with the derivations of and presentation of his equations. It was a rude awakening to realize the difficulty of the task given Maxwell's notation, because of the severe limits of conventional mathematical notation at the time of his work. The text, for me, is extremely difficult to read. Vector notation, with the exception of quaternions, had not come into usage at that time and few were aware of quaternions and less had any understanding or tolerance of them. With the aid of vector algebra and its notation (Heaviside and Gibbs), we can now express operations on the vector $\vec{B}$ in terms of its components B_x, B_y, B_z or $\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$, or further as some would prefer $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$. Maxwell knew the vector B consisted of three components that he labeled a, b, c and carried out his expression laboriously with each component $a, b, or c$ independently. Not only is this difficult and time-consuming to write, it is a nightmare to read and understand. With modern notation, the component name carries the vector identity, whereas with three different letters for the component names, not related to the vector name; it becomes taxing to keep track of what is what. This is especially difficult due to the complexity of the material that the reader is attempting to understand.

Below is an example from Maxwell's Treatise where he defines the vector components of the various physical quantities he is exploring.

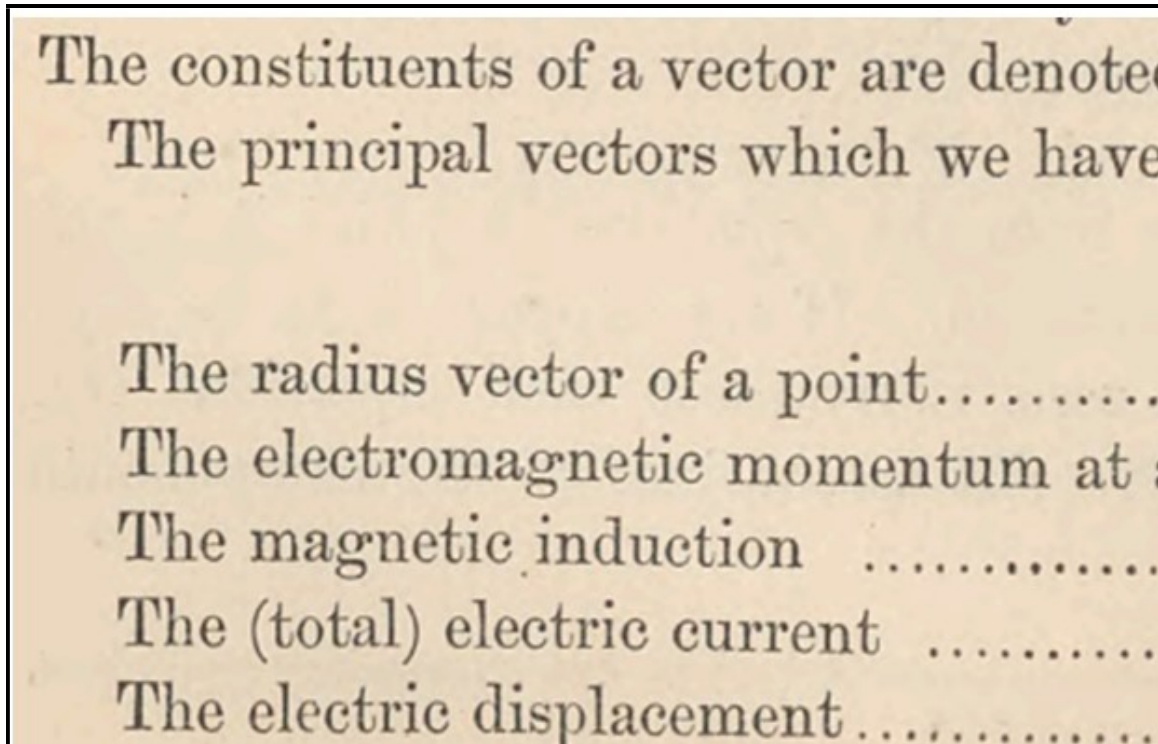


Figure 1 A Portion of Maxwell's Declaration of His Symbol Structure

A good example is of the translation I am working on is as follows from Article 598 in his Treatise

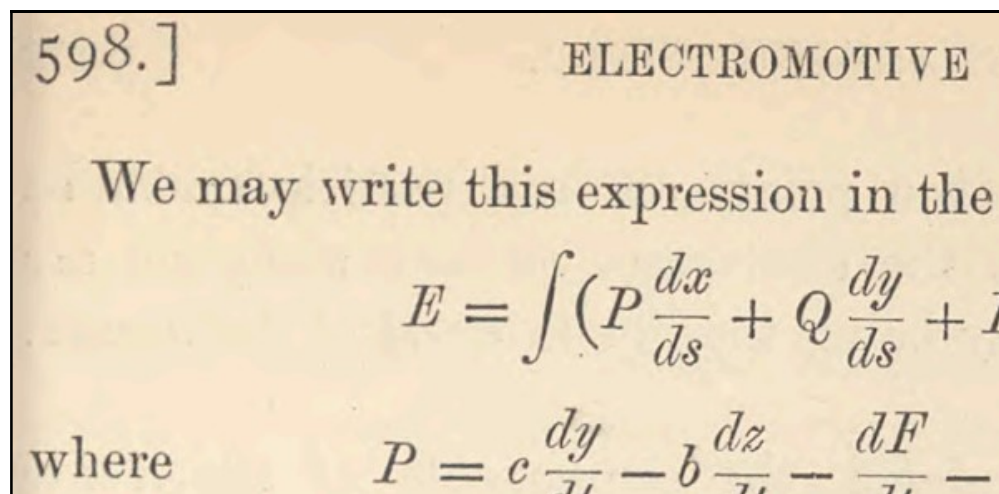


Figure 2 An Example of Maxwell's Presentation Using His Symbols

My Translation

We may write this expression in the form

$$V = \oint_l \left(E_x \frac{dy}{dl} - E_y \frac{dz}{dl} + E_z \frac{dx}{dl} \right)$$

where

$$\left[E_x = B_z \frac{dy}{dt} - B_y \frac{dz}{dt} - \frac{dA_x}{dt} - \frac{d\phi}{dx} \right]$$

Figure 3 My Translation Example of Maxwell's Presentation Using Modern (Heaviside's) Symbols

To me, obviously, it is much easier to comprehend and work with the formula in the translated form. There are a number of challenges to my work, some yet to be resolved, in interpreting just what Maxwell meant by a given expression, as he was not always consistent in naming and declaring variables and vectors, particularly in the vicinity of the area he is working. So, it was difficult to determine whether a given expression or variable was a *vector* or *scalar* quantity in his Treatise or an integral was open or closed. Sometimes issues are cleared up several pages downstream, sometimes not. Yet, some is still to be resolved. It is a tough, time consuming chore; however, there is fruit to be had.

I am now in a better position to converse with those asking the questions about Maxwell and quaternions. (After 20 years, they are no longer consulting me.) To add to that, in my view, Maxwell was not diligent in defining his variables prior to using them, or, at least, in the area that he was using them. (All this makes it more amazing to me that Oliver Heaviside, a self-taught individual, did read and understand what Maxwell had written, when most of his highly educated, contemporaries could not. Indeed, from this basis he created the vector algebra and notation in which to express Maxwell's work.) Trying, to read Maxwell's work and keeping track of the variables, and trying to find just what the particular



symbol relates to or what he is attempting to present is mind-boggling, to say the least. Another factor also comes into play here. Some comments from some of Maxwell's students go to the point here. They loved him dearly and were truly inspired by him, despite in their stated opinion, he was a “poor teacher”⁶.

Quaternions

Thus, Maxwell was quite limited in his expression of his ideas due to the primitive notation conventions of his time for vector algebra and calculus. Heaviside and Gibbs⁷ provided the really necessary leap forward in notation, that we engineers take for granted today, particularly vector algebra and notation. Not having the benefit of vector algebra and notation, Maxwell was quite hindered in attempting to share his ideas. It seems to me that Maxwell *envisioned* the benefit of vector notation brought forth in Hamilton's quaternions and latched on to the vector notation in quaternions and as stated in the Preliminary section of Volume I of his Treatise¹,

“10] *In distinguishing the kinds of physical quantities, it is of great importance to know how they are related to the directions of those coordinate axes that we usually employ in defining the positions of things. The introduction of coordinate axes into geometry by Des Cartes was one of the greatest steps in mathematical progress, for it reduced the methods of geometry to calculations performed on numerical quantities. The position of a point is made to depend on the length of three lines that are always drawn in determinate directions, and the line joining two points is in like manner considered as the resultant of three lines.*

But for many purposes in physical reasoning, as distinguished from calculation, it is desirable to avoid explicitly introducing the Cartesian coordinates, and to fix the mind at once on a point of space instead its three coordinates, and on the magnitude and direction of a force instead of its three components. This mode of contemplating geometrical and physical quantities is more primitive and more natural than the other, although the ideas connected with it did not receive their full development till Hamilton made the next great step in dealing with space, by the invention of his Calculus of Quaternions.

As the methods of Des Cartes are still the most familiar to students of science, and as they are really the most useful for purposes of calculation, we shall ex-



press all our results in Cartesian form. I am convinced; however, that the introduction of the ideas, as distinguished from the operations and methods of Quaternions, will be of great use to us in the study of all parts of our subject, and especially in electrodynamics, where we have to deal with a number of physical quantities, the relations of which to each other can be expressed far more simply by a few words of Hamilton, than by the ordinary equations."

I think what he saw in Hamilton's quaternions was the germ of what vector algebra could be and so as a result latched onto Hamilton's vector notation. In my translation to modern notation, I have not seen a single example of his carrying his implementation of quaternions any further than expressions in vector form "calling" those quaternions. ■ In fact, he was merely expressing the beginnings of vector notation.

The conclusion I *had* come to at this point in the process of my translation is that he expressed his equations in vector form and not quaternions because he may have thought he was expressing in quaternions. This is not to say that it may be possible to express Maxwell's equations in quaternions or some other advanced algebra to great advantage, I am just saying that I am suggesting that Maxwell did not. As far as I could tell, Maxwell did not carry out any quaternion algebra or calculation in his Treatise.

Now, let's look a little deeper. Just what are quaternions, anyway? In 1843 Hamilton introduced his quaternion algebra. A quaternion is a vector entity that also includes a *scalar* term. For those who are familiar with vector algebra, but not quaternions (see below), the added scalar really throws a wrench in the works. Following in the steps of Oliver Heaviside, I will without apology, present the following in my own notation, bringing with me considerable conventional notation.

Hamilton's original notation was quite innovative at the time; however, in my opinion, still muddies the water in current usage.

$$\boxed{a = a_0 + a_1 i + a_2 j + a_3 k}$$

Hamilton's Quaternion Form

and

$$\begin{aligned}
 i^2 &= j^2 = k^2 = ijk = -1 \\
 ij &= k = -ji \\
 jk &= i = -kj \\
 ki &= j = -ik
 \end{aligned}$$

First, I will use the four dot over-stroke to denote a quaternion, for example $\overset{\cdot\cdot\cdot\cdot}{A}$.

$$\begin{aligned}
 \overset{\cdot\cdot\cdot\cdot}{A} &= A_0 + A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \\
 &= A_0 + \vec{A}
 \end{aligned}$$

where

$$\begin{aligned}
 \hat{x}^2 &= \hat{y}^2 = \hat{z}^2 = \hat{x}\hat{y}\hat{z} = -1 \\
 \hat{x}\hat{y} &= \hat{z} = -\hat{y}\hat{x} \\
 \hat{y}\hat{z} &= \hat{x} = -\hat{z}\hat{y} \\
 \hat{z}\hat{x} &= \hat{y} = \hat{x}\hat{z}
 \end{aligned}$$

Hamilton used i, j, k for his unit vectors. I imagine that was because the notation of the over-caret for a unit vector had not evolved at that time. Something like $f(x)x$ to express the product of a function of x times the unit vector x is confusing and would suggest that $f(x)i$ would be much clearer *until* you could write $f(x)\hat{x}$. The i unit vector is actually for the x axis, so in my mind, $\hat{x}$ is a better choice for unit vector for the x axis than the unit vector i .

At last, it was becoming clear to me what Maxwell means by the scalar and vector products and it comes from quaternion multiplication. If you take two quaternion vectors $\overset{\cdot\cdot\cdot\cdot}{A}$ and $\overset{\cdot\cdot\cdot\cdot}{B}$ (In my notation the four over-dots define a quaternion, while the over arrow define a vector) whose components are

$$A_0, A_x, A_y, A_z, \text{ and } B_0, B_x, B_y, B_z \text{ or simply } (A_0 + \vec{A}) \text{ and } (B_0 + \vec{B})$$



are multiplied as follows

$$\begin{aligned}
 \vec{A} \vec{B} &= (A_0 + A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_0 + B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\
 &= (A_0 B_0 - A_x B_x - A_y B_y - A_z B_z) \\
 &\quad + A_0 (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) + B_0 (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \\
 &\quad + (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\
 &= A_0 B_0 - (A_x B_x + A_y B_y + A_z B_z) + A_0 \vec{B} + B_0 \vec{A} \\
 &\quad + (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\
 &= \underbrace{A_0 B_0 - \vec{A} \circ \vec{B}}_{\text{Scalar}} + \underbrace{A_0 \vec{B} + B_0 \vec{A} + \vec{A} \times \vec{B}}_{\text{Vector}}
 \end{aligned}$$

Thus the product of two quaternions is also a quaternion as it has a scalar and a vector part. Note that it also has the dot and cross products in the result.



Now if we take the quaternion product of the Del operator and the quaternion $\vec{B}$

$$\begin{aligned}
 \left(Del = \vec{\nabla} = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) = \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \right) \\
 \vec{\nabla} \vec{B} &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) (B_0 + B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\
 &= \left(\frac{\partial}{\partial x} B_x - \frac{\partial}{\partial y} B_y - \frac{\partial}{\partial z} B_z \right) \\
 &\quad + B_0 \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \\
 &\quad + \left(\frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y \right) \hat{i} + \left(\frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z \right) \hat{j} + \left(\frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \right) \hat{k} \\
 &= \left(\frac{\partial}{\partial x} B_x + \frac{\partial}{\partial y} B_y + \frac{\partial}{\partial z} B_z \right) + B_0 \vec{\nabla} \\
 &\quad + \left(\frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y \right) \hat{i} + \left(\frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z \right) \hat{j} + \left(\frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \right) \hat{k} \\
 &= \underbrace{\vec{\nabla} \circ \vec{B}}_{\text{Scalar}} + \underbrace{B_0 \vec{\nabla} + \vec{\nabla} \times \vec{B}}_{\text{Vector}} = \boxed{\underbrace{\vec{\nabla} \circ \vec{B}}_{\text{Scalar}} + \underbrace{\vec{\nabla} B_0 + \vec{\nabla} \times \vec{B}}_{\text{Vector}}}
 \end{aligned}$$

I will show my ignorance here in that there must be a quaternion version of Del, although this is the del used by Hamilton.

It is interesting to note that if $A_0 = B_0 = 0$ the quaternion product now becomes a vector product that results in a quaternion

$$\vec{\ddot{A}} \vec{\ddot{B}} \big|_{A_0=B_0=0} = \vec{A} \vec{B} = \boxed{- \underbrace{\vec{A} \circ \vec{B}}_{\substack{\text{Dot Product} \\ \text{(Scalar)}}} + \underbrace{\vec{A} \times \vec{B}}_{\substack{\text{Cross Product} \\ \text{(Vector)}}}}$$

and

$$\vec{\nabla} \vec{\ddot{B}} \big|_{A_0=B_0=0} = \vec{\nabla} \vec{B} = \boxed{- \underbrace{\vec{\nabla} \circ \vec{B}}_{\substack{\text{Divergence} \\ \text{(Scalar)}}} + \underbrace{\vec{\nabla} \times \vec{B}}_{\substack{\text{Curl} \\ \text{(Vector)}}}}$$

Thus the negative divergence (or Maxwell's convergence) and curl operators are the scalar and vector products of two quaternions whose scalar parts are zero.

To me, it is an interesting observation that vector algebra is actually a subset of quaternion algebra. If, in the quaternion “vectors” the scalars are all zero (What is the proper name for a quaternion vector? I have chosen to speak of them as qvectors.) They become and operate as ordinary vectors.

I have not been able to find a single incidence where Maxwell gives any variable both a scalar and a vector part. Thus, he did not express his electromagnetic equations in true quaternion form. There is one potential exception and that is; there is a distinct possibility and this pure is speculation, that there is potentially a case where the Maxwell's equations as stated are actually quaternions. For these equations to work as quaternions, the scalar components would have to be very small,  in the macro world. Because Maxwell's equations have shown their durability over the past century and a half and surely if the effects of these scalars existed, there would be some evidence. That's not to say that there has been some evidence and that it was ignored, as in quantum physics the effects of Plank's zero-point energy have been ignored. Just suppose that, that there are scalar values that could coexist with the apparent evidence, what would be required of them? If the scalar value magni-



tudes were on the order of the Plank constant, in other words in the vicinity of 10^{-34} , there would be no effect on the operation of Maxwell's equations stated as quaternions until the scale of the dimensions was in the order of the Planck constant. Then, a number of new variables come into play in Maxwell's quaternion equations as a result of products that were previously ignored and remain unidentified and unnamed. What would these new quantities be? Could they solve some of the mysteries of Quantum Mechanics? My knowledge of quantum mechanics is very limited, even more so than with integration and differentiation, as once I was reasonably adept at integration and differentiation. For example, for the product of two quaternions, the following products show up in addition to the dot and cross products

$$\vec{\ddot{A}} \vec{\ddot{B}} = \underbrace{A_0 B_0 + A_0 \vec{B} + B_0 \vec{A}}_{\text{Addition Products}} - \underbrace{\vec{A} \circ \vec{B}}_{\text{Dot Product}} + \underbrace{\vec{A} \times \vec{B}}_{\text{Cross Product}} .$$

And, for the gradient of a qvector

$$\vec{\nabla} \vec{\ddot{B}} = \underbrace{\vec{\nabla} B_0}_{\text{Addition Product}} - \underbrace{\vec{\nabla} \circ \vec{B}}_{\text{Divergence}} + \underbrace{\vec{\nabla} \times \vec{B}}_{\text{Curl}}$$

These could have new meaning in the quantum world and, again, it

might not. For example, just suppose $B_0 = \frac{k\hbar}{x}$, then at values of x approaching the value of the Planck constant $\hbar$, B_0 would have real meaning, yet be quite invisible in the macro world. This would also hold for the vector components if they contained such terms. What this means is that Maxwell's equations would transform dramatically in the quantum world. Whether that has any meaning, remains to be seen.

In reference to my first paragraph in this paper, I would call this **Maxwell's General Theory of Electromagnetics** if it proves fruitful. So be it!

Some Thoughts

After a lifetime, working with electromagnetics, I have some observations that to me are interesting. Most of us, in our professional life and hobbies such as ham radio, work with what I will call the *fruit* of past sci-



ence without examining the foundation of that fruit. I remember one time when working with a leading edge company, a Vice President that I respected and admired, stated with a slight undercurrent of derision about me, that “ ‘Ole Jim’ always goes back to first principles in his approach to problem solving”. While that was true for many things, it wasn’t true for Maxwell’s equations. His was an organization that was pushing the leading edge in Ethernet technology using many competent and powerful tools in their work. Working with the tools was more efficient and productive than coming from the basic science. That is true for the most part; however, I believe that you have get your feet back on the ground ever so often or you float off into gibberish.

A dear friend of mine asked why I was going to present this paper to QEX and not to one of the scientific journals. At the time, I didn’t have a good answer for him other than that was what I preferred. I have been a practicing engineer and a ham for better than 60 years. I have studied and experimented through-out that time. I have scant respect for formality and high platitudes. In addition to Maxwell and Heaviside, I highly admire Richard Feynman who ridiculed those ways quite effectively, while getting at and presenting the basics. “Planck's principle is the view that scientific change does not occur because individual scientists change their mind, but rather that successive generations of scientists have different views.” (Wikipedia) In other words, progress in science is measured in funerals. Formalism and pedantic control stifle innovation, while keeping some order. It is both a blessing and a curse.

QEX supports free thought and the expression of ideas and research in the ham community of which I am proud to be a part.

My thanks to Bob Karon AA6RK for his significant assistance in making my expression much clearer.

Notes and References

1. Maxwell, James Clerk, “A Treatise On Electricity & Magnetism”, Volume I, 1873, Oxford at the Clarendon Press, 1873
2. Maxwell, James Clerk, “A Treatise On Electricity & Magnetism”, Volume II, 1873, Oxford at the Clarendon Press, 1873
3. Oliver Heaviside reduced the equations presented by Maxwell in his Treatise to the four that we commonly today refer to as Maxwell’s Equations. Not once in my academic train-



ing do I remember the mention of Heaviside (or Gibbs) in relation to *Maxwell's* Equations. Much credit must be given to Heaviside and Gibb's translation of Maxwell's work in both the form and the creation of vector algebra, without which an understanding of Maxwell's work would have been delayed considerably.

4. Forbes, Nancy and Mahon, Basil, "Faraday, Maxwell and the Electromagnetic Field", 2014, Prometheus Books
5. Mahon, Basil "The Forgotten Genius of Oliver Heaviside", 2017, Prometheus Books
6. Paraphrased from pg 169 in ref 4.
7. Both Heaviside and Gibbs independently developed from their in-depth understanding of Maxwell's work the vector algebra and calculus we as engineers take for granted today. I must admit that I have not studied Gibb's work. (Josiah Willard Gibbs, 1839-1903, an American)

Bio

Jim Satterwhite, K4HJU, became interested in ham radio at age 12. Jim an ARRL Member, was first licensed in 1956 as KN4HJU and later that year as K4HJU, and is now an Amateur Extra class holder. He holds an FCC General Radiotelephone Operator's License, and is a registered Professional Engineer (Ret) in the state of North Carolina. He is an iNarte Certified EMC Engineer (Ret) and a Life Senior Member of IEEE. He received a BEE degree with High Honors from the University of Florida in 1965 and an MSEE degree from Purdue University in 1966. Jim has been involved in electronics research and development for the better part of 60 years, including 12 years as a member of the technical staff at Bell Labs and 34 years as a research and development engineer with Teltest Electronics, the company he founded in 1982. He holds a number of patents and patent applications. While in high school he built an AM transmitter from the ARRL Handbook. Jim spent a life time involved in electronics research and development, and in the design and construction of electronic equipment. Jim enjoys developing electronic systems and is more comfortable with MathCad, a VNA or a soldering iron than a microphone. "There is no greater teacher than the lab." His Amateur Radio interests have included slow scan TV in the early, early days, developing an antenna tuner using a unique power meter, and antenna analyzer design.

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