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GPS

MathCAD Position Solution

A Work in Progress

Rev 0.03
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1. Introduction

This document presents a Mathcad GPS position solution on one page in Mathcad. It is a very busy page. This Mathcad solution was written several years ago in preparation for writing a real-time proprietary differential GPS position solution in C for operation on an IBM PC. At that time, due to the speed of processors (Intel 486) and the number of equations requiring an evaluation, it was very cumbersome in Mathcad to make changes on page-one and then go to page three or more to see the result. Therefore, a successful attempt was made to place the entire solution on one page or screen in Mathcad. In that way, the results of changes could be viewed quickly, enhancing the development process.

Since that time, due to changes in later versions of Mathcad, the solution no longer fits all on one 8 1/2 x 11 sheet in Mathcad. Therefore, I have adjusted it to something a little larger. That one page sheet is attached at the end of this document in an orientation that will print well. It is also available a separate Adobe Acrobat pdf file. It is suggested that that file be printed and then use the figures in this document as a guide to the information on the printed figure, as the compressed figures in this document are barely readable.

2. GPS Position Solution

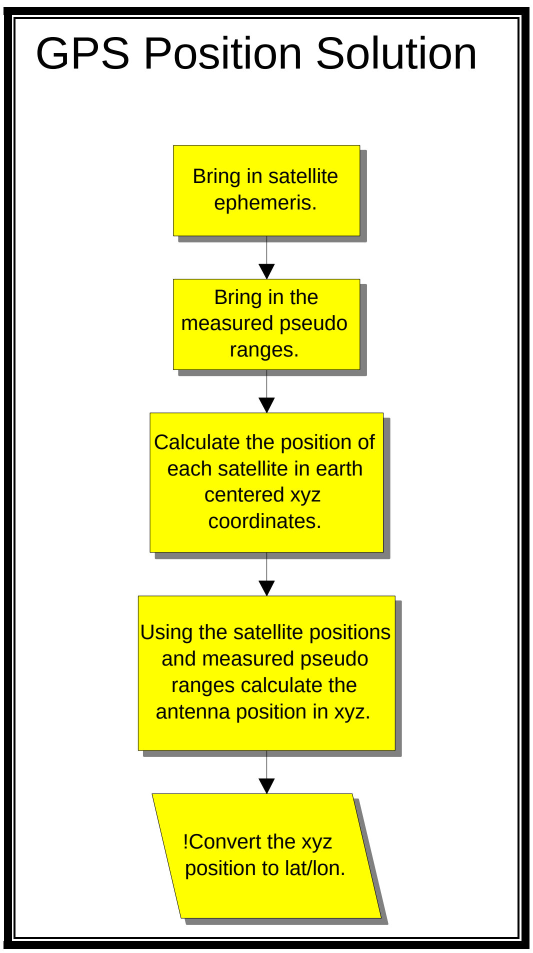

Figure 2-1 Position Solution Flow

3. One Page Solution

The one page solution is shown in Figure 3-1. This a busy figure and too compact as shown. See the attached figure or the separate file mentioned above for more clarity.

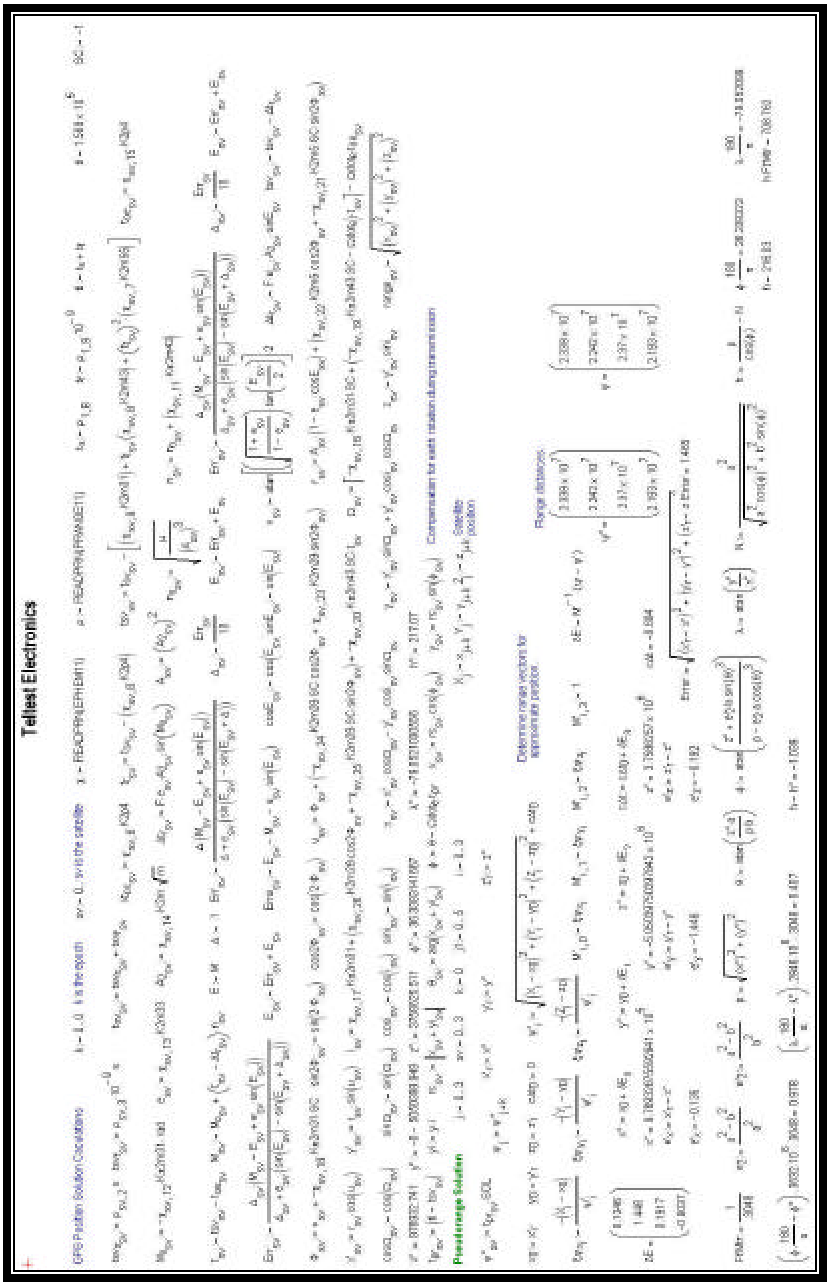

Figure 3-1 One Page Mathcad Solution

This difficult to view graphic is included here as a figure for continuity. See the attachment at the end of this document for a more detailed graphic.

4. Detail

Graphic figures are presented to the reader as a guide through the solution. Obviously, they are not clear. See the attachment at the end of this document for a more detailed graphic. The figures below are snapshots from that graphic.

4.1. Data Entry

Ephemeris and pseudorange data is brought in via data files as shown in Figure 4-1. EPHEM11 and PRANGE11 are two coherent data sets.

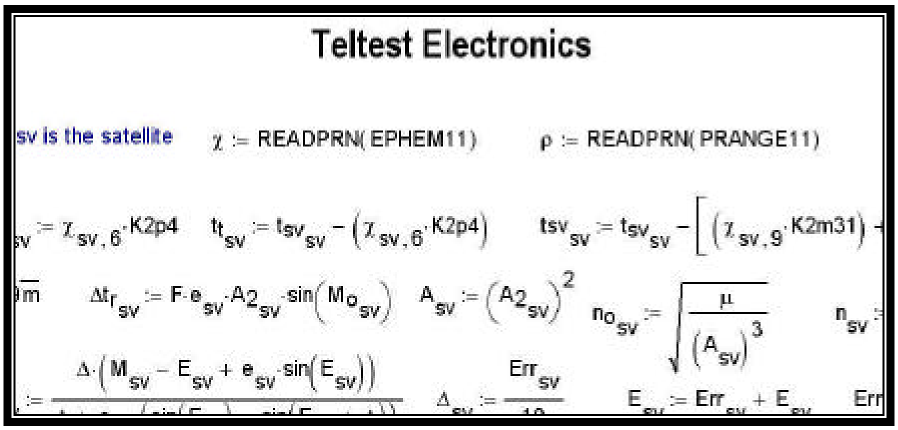

Figure 4-1 Bring Data Files In

4.2. Parse and Calculate Satellite Positions

The input data files are then parsed to obtain the specific variable needed for the calculations. The using these parsed variables the relevant GPS satellite positions are calculated in earth centered x y z coordinates.

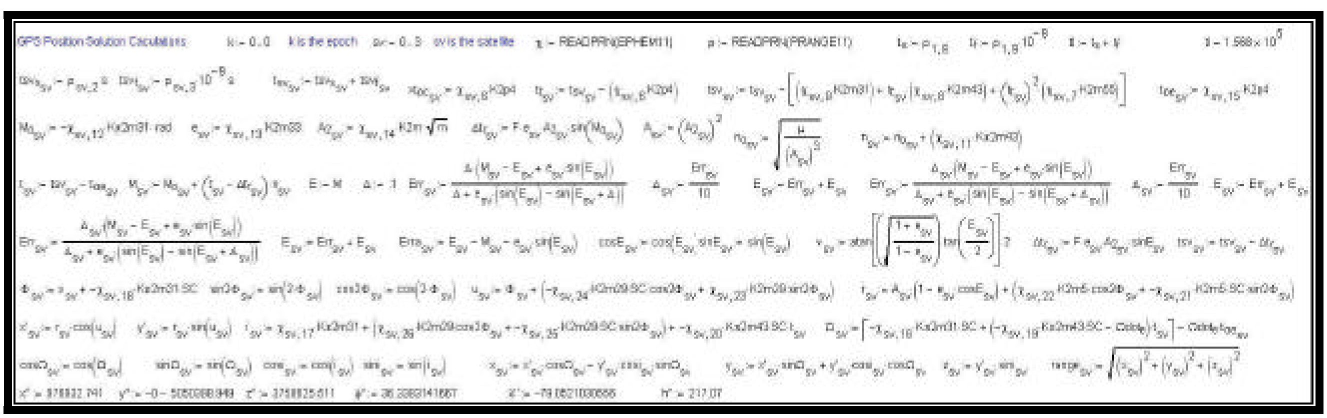

Figure 4-2 Parse Data Files and Calculate Satellite Positions

4.3. Calculate Antenna Position

Given the satellite positions, using the input pseudorange data, the receiver antenna position is calculated in earth centered x y z coordinates.

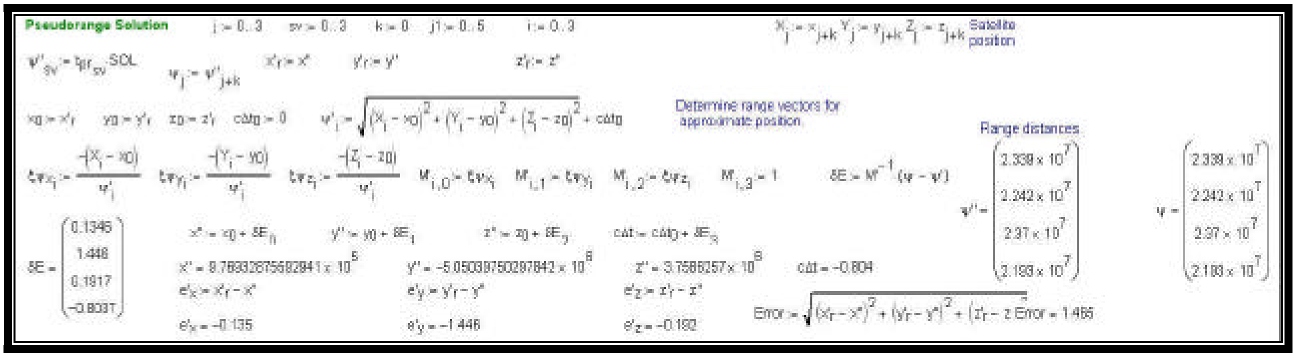

Figure 4-3 Calculate Antenna Position

4.4. Convert Position to Latitude, Longitude and Height

Finally, the position solution is converted to latitude, longitude and height above the geoid.

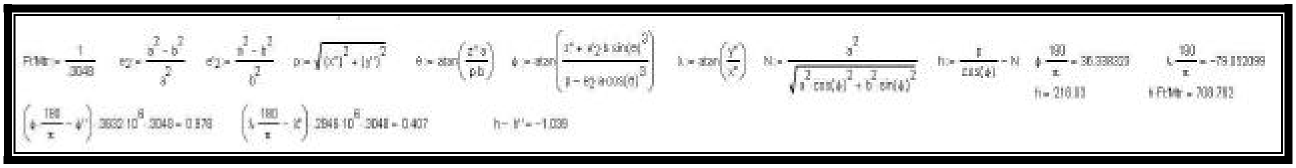

Figure 4-4 Convert x y z to Latitude/Longitude

5. The Future

There will be additions and clarifications to this document with time so the reader may want to look again later.

<u>End of Document</u>

GPS03.DOC

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GPS Position Solution Caculations $k := 0..0$ k is the epoch $sv := 0..$ sv is the satellite $\chi := READPRN(EPHEM11)$ $\rho := READPRN(PRANGE11)$ $t_s := \rho_{1,8}$ $t_f := \rho_{1,9} \cdot 10^{-9}$ $tl := t_s + t_f$ $tl = 1.568 \times 10^5$ $SC := -1$

$tsvs_{sv} := \rho_{sv,2}\ s$ $tsvf_{sv} := \rho_{sv,3} \cdot 10^{-9}\ s$ $tsv_{sv} := tsvs_{sv} + tsvf_{sv}$ $xtoc_{sv} := \chi_{sv,6} \cdot K2p4$ $t_{t_{sv}} := tsv_{sv} - (\chi_{sv,6} \cdot K2p4)$ $tsv_{sv} := tsv_{sv} - \left[(\chi_{sv,9} \cdot K2m31) + t_{t_{sv}} \cdot (\chi_{sv,8} \cdot K2m43) + (t_{t_{sv}})^2 \cdot (\chi_{sv,7} \cdot K2m55)\right]$ $toe_{sv} := \chi_{sv,15} \cdot K2p4$

$Mo_{sv} := -\chi_{sv,12} \cdot K\pi 2m31\ rad$ $e_{sv} := \chi_{sv,13} \cdot K2m33$ $A2_{sv} := \chi_{sv,14} \cdot K2m \sqrt{m}$ $\Delta tr_{sv} := F \cdot e_{sv} \cdot A2_{sv} \cdot \sin(Mo_{sv})$ $A_{sv} := (A2_{sv})^2$ $no_{sv} := \sqrt{\frac{\mu}{(A_{sv})^3}}$ $n_{sv} := no_{sv} + (\chi_{sv,11} \cdot K\pi 2m43)$

$t_{sv} := tsv_{sv} - toe_{sv}$ $M_{sv} := Mo_{sv} + (t_{sv} - \Delta tr_{sv}) \cdot n_{sv}$ $E := M$ $\Delta := .1$ $Err_{sv} := \frac{\Delta \cdot (M_{sv} - E_{sv} + e_{sv} \cdot \sin(E_{sv}))}{\Delta + e_{sv} \cdot (\sin(E_{sv}) - \sin(E_{sv} + \Delta))}$ $\Delta_{sv} := \frac{Err_{sv}}{10}$ $E_{sv} := Err_{sv} + E_{sv}$ $Err_{sv} := \frac{\Delta_{sv} \cdot (M_{sv} - E_{sv} + e_{sv} \cdot \sin(E_{sv}))}{\Delta_{sv} + e_{sv} \cdot (\sin(E_{sv}) - \sin(E_{sv} + \Delta_{sv}))}$ $\Delta_{sv} := \frac{Err_{sv}}{10}$ $E_{sv} := Err_{sv} + E_{sv}$

$Err_{sv} := \frac{\Delta_{sv} \cdot (M_{sv} - E_{sv} + e_{sv} \cdot \sin(E_{sv}))}{\Delta_{sv} + e_{sv} \cdot (\sin(E_{sv}) - \sin(E_{sv} + \Delta_{sv}))}$ $E_{sv} := Err_{sv} + E_{sv}$ $Erra_{sv} := E_{sv} - M_{sv} - e_{sv} \cdot \sin(E_{sv})$ $cosE_{sv} := \cos(E_{sv})$ $sinE_{sv} := \sin(E_{sv})$ $\nu_{sv} := atan\left[\left(\sqrt{\frac{1 + e_{sv}}{1 - e_{sv}}} \cdot \tan\left(\frac{E_{sv}}{2}\right)\right)\right] \cdot 2$ $\Delta tr_{sv} := F \cdot e_{sv} \cdot A2_{sv} \cdot sinE_{sv}$ $tsv_{sv} := tsv_{sv} - \Delta tr_{sv}$

$\Phi_{sv} := \nu_{sv} + -\chi_{sv,18} \cdot K\pi 2m31 \cdot SC$ $sin2\Phi_{sv} := \sin(2 \cdot \Phi_{sv})$ $cos2\Phi_{sv} := \cos(2 \cdot \Phi_{sv})$ $u_{sv} := \Phi_{sv} + (-\chi_{sv,24} \cdot K2m29 \cdot SC \cdot cos2\Phi_{sv} + \chi_{sv,23} \cdot K2m29 \cdot sin2\Phi_{sv})$ $r_{sv} := A_{sv} \cdot (1 - e_{sv} \cdot cosE_{sv}) + (\chi_{sv,22} \cdot K2m5 \cdot cos2\Phi_{sv} + -\chi_{sv,21} \cdot K2m5 \cdot SC \cdot sin2\Phi_{sv})$

$x'_{sv} := r_{sv} \cdot \cos(u_{sv})$ $y'_{sv} := r_{sv} \cdot \sin(u_{sv})$ $i_{sv} := \chi_{sv,17} \cdot K\pi 2m31 + (\chi_{sv,26} \cdot K2m29 \cdot cos2\Phi_{sv} + -\chi_{sv,25} \cdot K2m29 \cdot SC \cdot sin2\Phi_{sv}) + -\chi_{sv,20} \cdot K\pi 2m43 \cdot SC \cdot t_{sv}$ $\Omega_{sv} := \left[-\chi_{sv,16} \cdot K\pi 2m31 \cdot SC + (-\chi_{sv,19} \cdot K\pi 2m43 \cdot SC - \Omega dot_e) \cdot t_{sv}\right] - \Omega dot_e \cdot toe_{sv}$

$cos\Omega_{sv} := \cos(\Omega_{sv})$ $sin\Omega_{sv} := \sin(\Omega_{sv})$ $cosi_{sv} := \cos(i_{sv})$ $sini_{sv} := \sin(i_{sv})$ $x_{sv} := x'_{sv} \cdot cos\Omega_{sv} - y'_{sv} \cdot cosi_{sv} \cdot sin\Omega_{sv}$ $y_{sv} := x'_{sv} \cdot sin\Omega_{sv} + y'_{sv} \cdot cosi_{sv} \cdot cos\Omega_{sv}$ $z_{sv} := y'_{sv} \cdot sini_{sv}$ $range_{sv} := \sqrt{(x_{sv})^2 + (y_{sv})^2 + (z_{sv})^2}$

$x'' := 976932.741$ $y'' := -0 - 5050398.949$ $z'' := 3758625.511$ $\phi'' := 36.3383141667$ $\lambda'' := -79.0521030556$ $h'' := 217.07$

$tpr_{sv} := (tl - tsv_{sv})$ $yi := y \cdot i$ $rs_{sv} := |x_{sv} + yi_{sv}|$ $\theta_{sv} := arg(x_{sv} + yi_{sv})$ $\phi := \theta - \Omega dot_e \cdot tpr$ $x_{sv} := rs_{sv} \cdot \cos(\phi_{sv})$ $y_{sv} := rs_{sv} \cdot \sin(\phi_{sv})$ Compensation for earth rotation during transmission

Pseudorange Solution $j := 0..3$ $sv := 0..3$ $k := 0$ $j1 := 0..5$ $i := 0..3$ $X_j := x_{j+k}$ $Y_j := y_{j+k}$ $Z_j := z_{j+k}$ Satellite position

$\psi''_{sv} := tpr_{sv} \cdot SOL$ $\psi_j := \psi''_{j+k}$ $x'_r := x''$ $y'_r := y''$ $z'_r := z''$

$x_0 := x'_r$ $y_0 := y'_r$ $z_0 := z'_r$ $c\Delta t_0 := 0$ $\psi'_i := \sqrt{(X_i - x_0)^2 + (Y_i - y_0)^2 + (Z_i - z_0)^2} + c\Delta t_0$ Determine range vectors for approximate position.

Range distances.

$\xi\psi x_i := \frac{-(X_i - x_0)}{\psi'_i}$ $\xi\psi y_i := \frac{-(Y_i - y_0)}{\psi'_i}$ $\xi\psi z_i := \frac{-(Z_i - z_0)}{\psi'_i}$ $M'_{i,0} := \xi\psi x_i$ $M'_{i,1} := \xi\psi y_i$ $M'_{i,2} := \xi\psi z_i$ $M'_{i,3} := 1$ $\delta E := M'^{-1} \cdot (\psi - \psi')$

$\psi'' = \begin{pmatrix} 2.339 \times 10^7 \\ 2.242 \times 10^7 \\ 2.37 \times 10^7 \\ 2.193 \times 10^7 \end{pmatrix}$ $\psi = \begin{pmatrix} 2.339 \times 10^7 \\ 2.242 \times 10^7 \\ 2.37 \times 10^7 \\ 2.193 \times 10^7 \end{pmatrix}$

$\delta E = \begin{pmatrix} 0.1346 \\ 1.446 \\ 0.1917 \\ -0.8037 \end{pmatrix}$ $x'' := x_0 + \delta E_0$ $y'' := y_0 + \delta E_1$ $z'' := z_0 + \delta E_2$ $c\Delta t := c\Delta t_0 + \delta E_3$

$x'' = 9.76932875592941 \times 10^5$ $y'' = -5.05039750297842 \times 10^6$ $z'' = 3.7586257 \times 10^6$ $c\Delta t = -0.804$

$e'_x := x'_r - x''$ $e'_y := y'_r - y''$ $e'_z := z'_r - z''$ $Error := \sqrt{(x'_r - x'')^2 + (y'_r - y'')^2 + (z'_r - z''...}$ $Error = 1.465$

$e'_x = -0.135$ $e'_y = -1.446$ $e'_z = -0.192$

$Ft'Mtr := \frac{1}{.3048}$ $e_2 := \frac{a^2 - b^2}{a^2}$ $e'_2 := \frac{a^2 - b^2}{b^2}$ $p := \sqrt{(x'')^2 + (y'')^2}$ $\theta := atan\left(\frac{z'' \cdot a}{p \cdot b}\right)$ $\phi := atan\left(\frac{z'' + e'_2 \cdot b \cdot \sin(\theta)^3}{p - e_2 \cdot a \cdot \cos(\theta)^3}\right)$ $\lambda := atan\left(\frac{y''}{x''}\right)$ $N := \frac{a^2}{\sqrt{a^2 \cdot \cos(\phi)^2 + b^2 \cdot \sin(\phi)^2}}$ $h := \frac{p}{\cos(\phi)} - N$ $\phi \cdot \frac{180}{\pi} = 36.338323$ $\lambda \cdot \frac{180}{\pi} = -79.052099$

$h = 216.03$ $h \cdot Ft'Mtr = 708.762$

$\left(\phi \cdot \frac{180}{\pi} - \phi''\right) \cdot .3632 \cdot 10^6 \cdot .3048 = 0.978$ $\left(\lambda \cdot \frac{180}{\pi} - \lambda''\right) \cdot .2946 \cdot 10^6 \cdot .3048 = 0.407$ $h - h'' = -1.039$